Basic Integration
Test review
answer Key

(1) $\int (2x^3 - 5x + 2)\,dx$

$$\boxed{\tfrac{1}{2}x^4 - \tfrac{5}{2}x^2 + 2x + C}$$

(2) $\int \dfrac{4x^3 + 5x^2 + 8x}{x}\,dx$

$\int (4x^2 + 5x + 8)\,dx$

$$\boxed{\tfrac{4}{3}x^3 + \tfrac{5}{2}x^2 + 8x + C}$$

(3) $\int (3x\sqrt{x} + 2)\,dx$

$\int (3x^{\frac{3}{2}} + 2)\,dx$

$$\boxed{\tfrac{6}{5}x^{5/2} + 2x + C}$$

(4) $\int \dfrac{5x^3 + 4x^{\frac{2}{3}} - 2x\sqrt[3]{x}}{3x^{\frac{1}{2}}}\,dx$

$\int \left(\tfrac{5}{3}x^{\frac{5}{2}} + \tfrac{3}{4}x^{\frac{1}{6}} - \tfrac{2}{3}x^{\frac{5}{6}}\right)dx$

$$\boxed{\tfrac{10}{21}x^{7/2} + \tfrac{9}{14}x^{\frac{7}{6}} - \tfrac{4}{11}x^{\frac{11}{6}} + C}$$

(5) $\int \tan(x)\,dx$

$$\boxed{-\ln|\cos(x)| + C}$$

(6) $\int \sqrt{3x+5}\,dx$

$u = 3x + 5$
$du = 3\,dx$
$dx = \dfrac{du}{3}$

$\int u^{\frac{1}{2}}\,\dfrac{du}{3}$

$\tfrac{1}{3}\int u^{\frac{1}{2}}\,du$

$\tfrac{1}{3}\left[\tfrac{2}{3}u^{\frac{3}{2}}\right] + C$

$$\boxed{\tfrac{2}{9}(3x+5)^{\frac{3}{2}} + C}$$

(7) $\int 2e^x\,dx$

$2\int e^x\,dx$

$$\boxed{2e^x + C}$$

(8) $$\int e^{4x-1}\, dx$$

$u = 4x-1 \quad du = 4\, dx$

$dx = \frac{du}{4}$

$$\int e^{u}\, \frac{du}{4} = \frac{1}{4}\int e^{u} du$$

$$= \frac{1}{4} e^{u} + c = \boxed{\frac{1}{4} e^{4x-1} + c}$$

(9) $$\int \cos^2(3x)\sin(3x)\, dx$$

$u = \cos(3x) \quad du = -3\sin(3x)\, dx$

$dx = \frac{du}{-3\sin(3x)}$

$$\int u^2 \sin(3x)\, \frac{du}{-3\sin(3x)}$$

$$-\frac{1}{3}\int u^2 du = -\frac{1}{3}\left[\frac{1}{3}u^3\right] + c$$

$$\boxed{= -\frac{1}{9}\cos^3(3x) + c}$$

(10) $$\int 5x\sqrt{3x^2+5}\, dx$$

$u = 3x^2+5 \quad du = 6x\, dx$

$dx = \frac{du}{6x}$

$$\int 5x\, u^{\frac{1}{2}}\, \frac{du}{6x}$$

$$\frac{5}{6}\int u^{\frac{1}{2}} du$$

$$\frac{5}{6}\left[\frac{2}{3}u^{\frac{3}{2}}\right] + c$$

$$\boxed{\frac{5}{9}(3x^2+5)^{\frac{3}{2}} + c}$$

(11) $$\int_0^3 (2x^3-5)\, dx$$

$$\frac{1}{2}x^4 - 5x\Big|_0^3$$

$$\left[\frac{1}{2}(3)^4 - 5(3)\right] - \left[\frac{1}{2}(0)^4 - 5(0)\right]$$

$$\frac{81}{2} - 15 - 0$$

$$\boxed{\frac{51}{2}}$$

(12) $$\int_0^{\pi} \operatorname{Tan}(x)\,dx$$

$$-\ln|\cos(x)|\Big|_0^{\pi}$$

$$-\ln|\cos(\pi)| + \ln|\cos(0)|$$

$$-\ln|-1| + \ln|1|$$

$$-\ln(1) + \ln(1) = \boxed{0}$$

(13) $$\int_2^5 x\sqrt{x^2+3}\,dx$$

$u = x^2+3 \quad du = 2x\,dx$

$$dx = \frac{du}{2x}$$

$$\int_7^{28} \cancel{x}\, u^{\frac{1}{2}} \frac{du}{2\cancel{x}}$$

$$\frac{1}{2}\int_7^{28} u^{\frac{1}{2}}\,du = \frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}\Big|_7^{28}\right]$$

$$= \frac{1}{2}\left[\frac{2}{3}(28)^{\frac{3}{2}} - \frac{2}{3}(7)^{3/2}\right]$$

$$= \frac{1}{2}[86.4279] = \boxed{43.214}$$

(14) $$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin(2x)\,dx$$

$$-\frac{1}{2}\cos(2x)\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

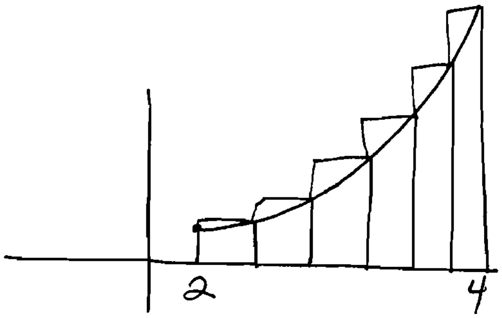
$$-\frac{1}{2}\cos\left(\cancel{2}\frac{3\pi}{\cancel{2}}\right) - -\frac{1}{2}\cos\left(\cancel{2}\frac{\pi}{\cancel{2}}\right)$$

$$-\frac{1}{2}(-1) + \frac{1}{2}(-1) =$$

$$\frac{1}{2} - \frac{1}{2} = 0$$

(15)

a)



$\Delta x = \frac{4-2}{6} = \frac{1}{3}$

Rt End pt $= 2 + \frac{1}{3}i = \frac{6+i}{3}$

height $= \left(\frac{6+i}{3}\right)^2 - 3$

$$A \approx \frac{1}{3} \sum_{i=1}^{6} \left(\frac{6+i}{3}\right)^2 - 3 = \boxed{14.7037 \ u^2}$$

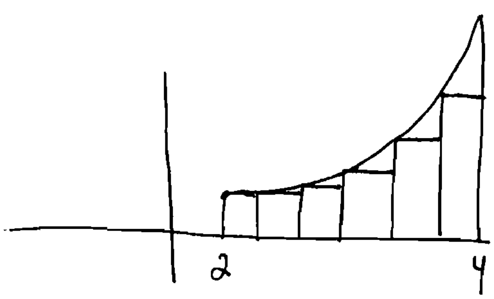
— or —

$\Delta x = \frac{1}{3}$ Rt End pts $= \frac{7}{3}, \frac{8}{3}, 3, \frac{10}{3}, \frac{11}{3}, 4$

$$A \approx \frac{1}{3}\left[f\left(\tfrac{7}{3}\right) + f\left(\tfrac{8}{3}\right) + f(3) + f\left(\tfrac{10}{3}\right) + f\left(\tfrac{11}{3}\right) + f(4)\right]$$

$$A \approx \boxed{14.7037 \ u^2}$$

b)



$\Delta x = \frac{1}{3}$

Lt End pts $= 2 + \left(\frac{1}{3}(i-1)\right) = 2 + \frac{i-1}{3} = \frac{i+5}{3}$

height $= \left(\frac{i+5}{3}\right)^2 - 3$

$$A \approx \frac{1}{3} \sum_{i=1}^{6} \left(\frac{i+5}{3}\right)^2 - 3 = \boxed{10.7037 \ u^2}$$

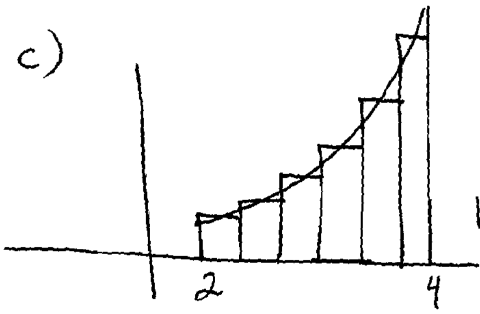
— or —

$\Delta x = \frac{1}{3}$ Left end pts $= 2, \frac{7}{3}, \frac{8}{3}, \frac{9}{3}, \frac{10}{3}, \frac{11}{3}$

$$A \approx \frac{1}{3}\left[f(2) + f\left(\tfrac{7}{3}\right) + f\left(\tfrac{8}{3}\right) + f(3) + f\left(\tfrac{10}{3}\right) + f\left(\tfrac{11}{3}\right)\right]$$

$$A \approx \boxed{10.7037 \ u^2}$$

(15) c)



$\Delta x = \frac{1}{3}$

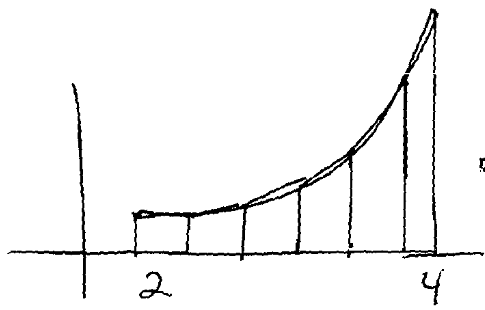
mid pts: $\frac{13}{6}, \frac{15}{6}, \frac{17}{6}, \frac{19}{6}, \frac{21}{6}$

height: $f(\frac{13}{6}), f(\frac{15}{6}), f(\frac{17}{6}), f(\frac{19}{6}), f(\frac{21}{6})$

$$A \approx \frac{1}{3}\left[f(\tfrac{13}{6}) + f(\tfrac{15}{6}) + f(\tfrac{17}{6}) + f(\tfrac{19}{6}) + f(\tfrac{21}{6})\right]$$

$$A \approx \boxed{12.6482\ u^2}$$

(15) d)



$\Delta x = \frac{1}{3}$

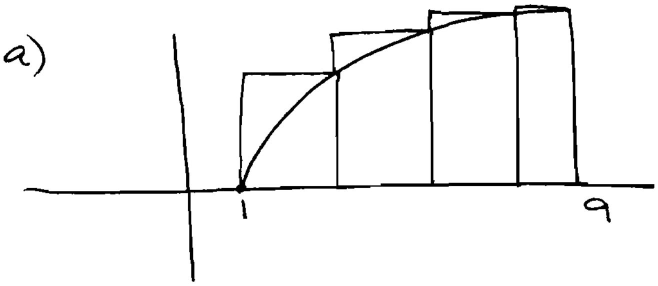
End pts:

$2, \frac{7}{3}, \frac{8}{3}, 3, \frac{10}{3}, \frac{11}{3}, 4$

$$A \approx \frac{1}{3}\left[\frac{1}{2}\left(f(2) + 2f(\tfrac{7}{3}) + 2f(\tfrac{8}{3}) + 2f(3) + 2f(\tfrac{10}{3}) + 2f(\tfrac{11}{3}) + f(4)\right)\right]$$

$$A \approx \boxed{12.7037\ u^2}$$

16 a)



$\Delta x = \frac{9-1}{4} = 2$

REP: $1 + 2i$

height: $-1 + \sqrt{1+2i}$

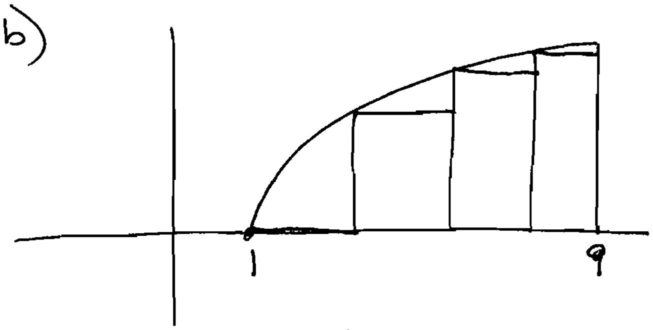
$$A \approx 2 \sum_{i=1}^{4} \left(-1 + \sqrt{1+2i}\right) = \boxed{11.2277 \, u^2}$$

— or —

$\Delta x = 2$ REP: 3, 5, 7, 9

$$A \approx 2\left[f(3) + f(5) + f(7) + f(9)\right] = \boxed{11.2277 \, u^2}$$

b)



$\Delta x = 2$

LEP: $1 + 2(i-1) = 2i - 1$

height: $-1 + \sqrt{2i-1}$

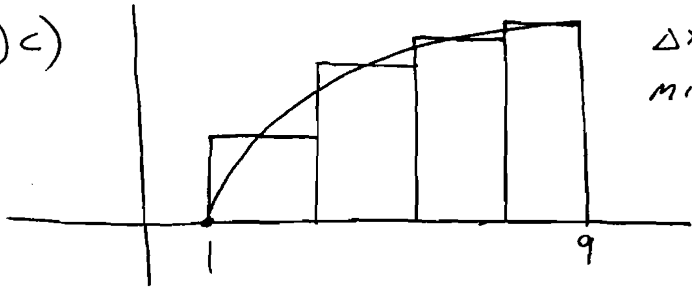
$$A \approx 2 \sum_{i=1}^{4} \left(-1 + \sqrt{2i-1}\right) = \boxed{7.2277 \, u^2}$$

— or —

$\Delta x = 2$ LEP: 1, 3, 5, 7

$$A = 2\left[f(1) + f(3) + f(5) + f(7)\right] = \boxed{7.2277 \, u^2}$$

16 c)

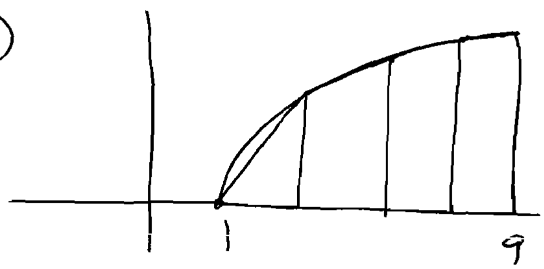


$\Delta x = 2$

mid pts: 2, 4, 6, 8

$$A \approx 2\left[f(2) + f(4) + f(6) + f(8)\right] = \boxed{9.3843\ u^2}$$

d)



$\Delta x = 2$

End pts: 1, 3, 5, 7, 9

$$A \approx \frac{2}{2}\left[f(1) + 2f(3) + 2f(5) + 2f(7) + f(9)\right]$$

$$A \approx \boxed{9.2277\ u^2}$$

17 $f(x) = x^3 + x^2 - 6x \quad [0, 3]$

$$x^3 + x^2 - 6x = 0$$

$$x(x^2 + x - 6) = 0$$

$$x(x+3)(x-2) = 0$$

$$x = 0, -3, 2$$

$x = 2$

$$\left|\int_0^2 (x^3 + x^2 - 6x)\,dx\right| + \int_2^3 (x^3 + x^2 - 6x)\,dx$$

$$\left|\frac{1}{4}x^4 + \frac{1}{3}x^3 - 3x^2 \Big|_0^2\right| + \frac{1}{4}x^4 + \frac{1}{3}x^3 - 3x^2 \Big|_2^3$$

$$\left|-\frac{16}{3}\right| + \frac{91}{12} = \boxed{\frac{155}{12}\ u^2}$$

$$\boxed{12.9167\ u^2}$$

alternative set up: (for calc section)

$$\int_0^3 \left|x^3 + x^2 - 6x\right|\,dx$$

(18)

$$f(x) = \sqrt{2x-5} - 3 \quad [3, 15]$$

$$\sqrt{2x-5} - 3 = 0$$

$$\sqrt{2x-5} = 3$$

$$2x - 5 = 9$$

$$x = 7$$

$$\left| \int_3^7 \left(\sqrt{2x-5} - 3\right) dx \right| + \int_7^{15} \left(\sqrt{2x-5} - 3\right) dx$$

$$\left| \frac{-10}{3} \right| + \frac{26}{3} = \boxed{12u^2}$$

set up on calc section:

$$\int_3^{15} \left| \sqrt{2x-5} - 3 \right| dx$$

(19) $f(x) = \frac{3}{x} \quad [e, e^2]$

$$3\int_e^{e^2} \frac{1}{x}\, dx = 3\left[\ln|x| \Big|_e^{e^2} \right] =$$

$$3\left[\ln(e^2) - \ln e \right] = 3\left[2\ln(e) - \ln(e) \right]$$

$$3\left[2(1) - 1 \right] = \boxed{3u^2}$$

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(20) $f(x) = \cos(x) \quad \left[0, \frac{3\pi}{2}\right]$

$$\cos(x) = 0$$

$$x = \frac{\pi}{2}$$

$$\int_0^{\pi/2} \cos(x)\, dx + \left| \int_{\pi/2}^{3\pi/2} \cos(x)\, dx \right|$$

$$\sin(x) \Big|_0^{\frac{\pi}{2}} + \left| \sin(x) \Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \right|$$

$$\sin\left(\frac{\pi}{2}\right) - \sin(0) + \left| \sin\left(\frac{3\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right) \right|$$

$$1 - 0 + \left| -1 - 1 \right|$$

$$1 + 2 = \boxed{3u^2}$$

(21) $V(t) = 3t^2 - 2t - 8$

$V(t) = 0$

$t = 2$

$$\left| \int_0^2 V(t)\,dt \right| + \int_2^4 V(t)\,dt$$

$$12 + 28 = \boxed{40\text{ ft}}$$

(22) $\int_0^4 V(t)\,dt = \boxed{16\text{ ft}}$

(23) $a(t) = 6t - 2$

$a(3) = 6(3) - 2 = \boxed{16\text{ ft/sec}^2}$

(24)

$$\left| \int_0^2 V(t)\,dt \right| + \int_2^3 V(t)\,dt$$

$$12 + 6 = \boxed{18\text{ ft}}$$

(25) $\int_0^3 V(t)\,dt = \boxed{-6\text{ ft}}$

(26) $a(1) = 6(1) - 2 = \boxed{4\text{ ft/sec}^2}$

(27) $\frac{1}{4}\int_0^4 V(t)\,dt = \boxed{4\text{ ft/sec}}$

$V(t) = t^3 - 3t^2 - 4t$

28 $\int_0^5 |v(t)|\,dt = \boxed{45.25 \text{ ft}}$

*you could have also split the integral at $t = 4$

29 $\int_0^5 v(t)\,dt = \boxed{-18.75 \text{ ft}}$

30 $a(t) = 3t^2 - 6t - 4$

$a(2) = 3(2)^2 - 6(2) - 4$

$12 - 12 - 4 = \boxed{-4 \text{ ft/sec}^2}$

31 $\int_0^3 |v(t)|\,dt = \boxed{24.75 \text{ ft}}$

32 $\int_0^3 v(t)\,dt = \boxed{-24.75 \text{ ft}}$

33 $a(4) = \boxed{20 \text{ ft/sec}^2}$

34 $\frac{1}{5}\int_0^5 v(t)\,dt = \boxed{-3.75 \text{ ft/sec}}$

35 $\frac{1}{4}\int_0^4 a(t)\,dt = \boxed{0 \text{ ft/sec}^2}$ — or — $\frac{V(4) - V(0)}{4 - 0} = \frac{0}{4} = \boxed{0 \text{ ft/sec}^2}$

36 $S(t) = \frac{1}{4}t^4 - t^3 - 2t^2 + C$

$15 = \frac{1}{4}(2)^4 - 2^3 - 2(2)^2 + C$

$15 = 4 - 8 - 8 + C$

$C = 27$

$S(t) = \frac{1}{4}t^4 - t^3 - 2t^2 + 27$

$\rightarrow S(4) = \boxed{-5 \text{ ft}}$

37 $\int_2^{2x^2-3} (x^2 + 3x - 1)\,dx =$

$\boxed{4x\left[(2x^2-3)^2 + 3(2x^2-3) - 1\right]}$